PHYS 301
Electricity and Magnetism

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Today!

- Integral vector calculus:
  - Fundamental theorem of curls
  - Fundamental theorem of divergences

Integral Calculus: Curls

- The **fundamental theorem of curls**:

$$\int_{\text{surface}} \left(\vec{\nabla}\times\vec{V}\right)\cdot d\vec{A} = \oint_{\text{line}} \vec{V}\cdot d\vec{l}$$

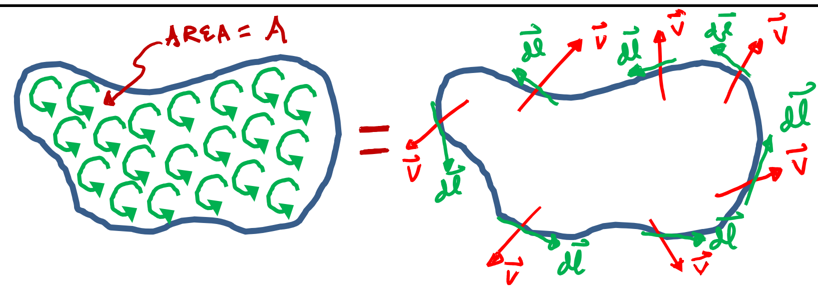
- Also called ***STOKES' THEOREM***
- NOTE:

$$\oint\left(\vec{\nabla}\times\vec{V}\right)\cdot d\vec{A} = 0 \;\; \forall \text{ closed surfaces}$$

$$\int\left(\vec{\nabla}\times\vec{V}\right)\cdot d\vec{A} \text{ depends only on the boundary; not on the surface!}$$

e.g., soap bubble!

2D Interpretation:



$$\int_{\text{SURF}} \underbrace{(\vec{\nabla}\times\vec{V})\cdot d\vec{A}}_{\text{CURL AT GIVEN POINT WITHIN AREA, A.}} = \oint_{\text{LINE}} \underbrace{\vec{V}\cdot d\vec{l}}_{\text{COMPONENT OF } \vec{V} \parallel d\vec{l}}$$

Direction of integration of ***LINE INTEGRAL*** *around boundary*
Is used to define ***POSITIVE*** *direction for* $d\vec{A}$ *: use* ***R H Rule****!*

(CURL FINGERS IN DIRECTION OF $d\vec{l}$, THUMB GIVES DIRECTION OF $+d\vec{A}$)

Integral Calculus: Divergences

- The **fundamental theorem of divergences**:

$$\int_{\text{vol}} \left(\vec{\nabla} \cdot \vec{B}\right) dV = \oint_{\text{surface}} \vec{B} \cdot d\vec{A}$$

- Also called ***GAUSS' THM*** or ***GREEN's THM***
- NOTE: $d\vec{A} = \hat{n}\, dA$ where $\hat{n}$ is $\perp$ the surface, pointing outward from the enclosed volume.

  $\vec{B} \cdot \hat{n} =$ component of $\vec{B} \perp$ surface.